Overview

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2. A motivating example from my own research
3. Crack growth example
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5. A last example - Street magician
Development of books

1989

Generalized Linear Models
SECOND EDITION

P. McCullagh and J.A. Nelder

2001

IN ALL LIKELIHOOD

Yudi Pawitan

2006

Generalized Linear Models with Random Effects
Unified Analysis via H-likelihood

Younjo Lee
John A. Nelder
Yudi Pawitan

2017

DATA ANALYSIS USING HIERARCHICAL GENERALIZED LINEAR MODELS WITH R

Younjo Lee
Lars Rönnegård
Menegosk Noh
Regression models using various distributions

**Gaussian**

\[
L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(y_i - \mu)^2}{2\sigma^2}}
\]

\[
\mu = X\beta
\]

**Poisson**

\[
L = \prod_{i=1}^{n} \frac{\mu^{y_i} e^{-\mu}}{y_i!}
\]

\[
\log(\mu) = X\beta
\]

**Binomial**

\[
L = \prod_{i=1}^{n} \binom{n}{y_i} \mu^{y_i} (1 - \mu)^{n-y_i}
\]

\[
\logit(\mu) = X\beta
\]

**Gamma**

\[
L = \prod_{i=1}^{n} \frac{1}{\Gamma(k)} \frac{\theta^k}{y_i^{k-1}} e^{-\frac{\theta}{y_i}}
\]

\[
k\theta \equiv \mu; \ 1/\mu = X\beta
\]
Generalized Linear Models

GLM

Common estimation algorithm using iterative regression
- Fast and easy to implement
- *Linear regression model checking tools***

**Gaussian**

\[
L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{1}{2\sigma^2}(y_i-\mu)^2}
\]

\[\mu = X\beta\]

**Poisson**

\[
L = \prod_{i=1}^{n} \frac{\mu^{y_i} e^{-\mu}}{y_i!}
\]

\[\log(\mu) = X\beta\]

**Binomial**

\[
L = \prod_{i=1}^{n} \binom{n}{y_i} \mu^{y_i} (1 - \mu)^{n-y_i}
\]

\[\logit(\mu) = X\beta\]

**Gamma**

\[
L = \prod_{i=1}^{n} \frac{1}{\Gamma(k)} \frac{1}{\theta^k} y_i^{k-1} e^{-\frac{y_i}{\theta}}
\]

\[k\theta \equiv \mu; \ 1/\mu = X\beta\]

Lee, Rönnegård & Noh
A model checking plot in linear regression

\[ y = X\beta + e \]
Hierarchical Generalized Linear Models


GLM approach for fitting
- Linear mixed models
- Generalized linear mixed models (Laplace approximation)
- Mixed models with non-Gaussian random effects
- Above models + dispersion modelling
Hierarchical Generalized Linear Models

Linear model with random effects

\[ y = X\beta + Zu + e \]

\[ u \sim N(0, \sigma_u^2) \]

\[ e \sim N(0, \sigma_e^2) \]

Random effects

- Repeated measurements (on the same subject) the most common application
- The parameter is assumed to be sampled from a population of parameters (not fixed)
- The variance component \( \sigma_u^2 \) is of interest
- Estimating \( \hat{u} \) can be of interest for: prediction (including spatial Kriging) and ranking subjects.
Hierarchical Generalized Linear Models

Linear model

\[ y = X\beta + e \]
\[ e \sim N(0, \sigma^2_e) \]

Linear model including a dispersion model

\[ y = X\beta + e \]
\[ e \sim N(0, \exp(X_d\beta_d)) \]
Hierarchical Generalized Linear Models

- **Linear model**
- **Linear mixed model (LMM)**
- **Generalized linear model (GLM)**
- **Generalized linear mixed model (GLMM)**
- **Joint GLM**
  - Generalized linear model including dispersion model with fixed effects
- **Hierarchical GLM (HGLM)**
  - Generalized linear model including Gaussian and/or non-Gaussian random effects. Dispersion can be modelled using fixed effects.
Hierarchical Generalized Linear Models

- Linear model
- Linear mixed model (LMM)
- Generalized linear model (GLM)
- Joint GLM
  - Generalized linear mixed model (GLMM)
    - Generalized linear model including Gaussian random effects
- Hierarchical GLM (HGLM)
  - Generalized linear model including Gaussian and/or non-Gaussian random effects. Dispersion can be modelled using fixed effects.
- Frailty HGLM
  - HGLMs for survival analysis including competing risk models
- Double HGLM (DHGLM)
  - HGLM including dispersion model with both fixed and random effects
- Factor analysis
- Structural Equation Models (SEM)
- HGLMs with correlated random effects
  - Including spatial, temporal correlations, splines, GAM.
A motivating example from my own research

Animals ranked by their genetic potential given by the estimated random effects in a linear mixed model

\[ y = X\beta + Za + e \]

The random effects \( a \sim N(0, \sigma_a^2 A) \) have a correlation matrix \( A \) computed from pedigree information (see e.g., Chapter 17 of Pawitan (2001)).
A motivating example from my own research

Animals ranked by their genetic potential given by the estimated random effects in a linear mixed model

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The random effects \( a \sim N(0, \sigma_a^2 A) \) have a correlation matrix \( A \) computed from pedigree information (see e.g., Chapter 17 of Pawitan (2001)).

\( \hat{a} = \) estimated breeding values
A motivating example from my own research

Animals ranked by their genetic potential given by the estimated random effects in a linear mixed model

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The random effects \( a \sim N(0, \sigma_a^2 A) \) have a correlation matrix \( A \) computed from pedigree information (see e.g., Chapter 17 of Pawitan (2001)).

\( \hat{a} = \) estimated breeding values

Possible to estimate breeding values for the variance!
Distributions of observed litter sizes and the number of observations per sow.

Figure 1.4 Distributions of observed litter sizes and the number of observations per sow.

\[ y = X\beta + Za + e. \]
A motivating example from my own research
Crack growth data

Hudak et al. (1978) crack lengths measured on a compact tension steel. 21 metallic specimens, crack lengths recorded every 104 cycles

- $y =$ increment of crack length
- $\text{crack0} =$ covariate for the mean part of the model
- $\text{cycle} =$ covariate for the dispersion part of the model
Crack growth data
Crack growth - Using the \texttt{hglm} package

\begin{verbatim}
res_glm <- glm(y ~ crack0, family = Gamma(link=log), data = data_crack_growth )
\end{verbatim}
Crack growth - Using the **hglm** package

```r
res_glm <- glm(y ~ crack0, family= Gamma(link=log),
               data= data_crack_growth )

library(hglm)
## GLMM ##
res_glmm <- hglm2(y ~ crack0 + (1|specimen), family= Gamma(link=log),
                  data= data_crack_growth )
```

Lee, Rönnegård & Noh
HGLM book
Crack growth - Using the **hglm** package

```r
res_glm <- glm(y ~ crack0, family= Gamma(link=log),
              data= data_crack_growth )

library(hglm)
## GLMM ##
res_glmm <- hglm2(y ~ crack0 + (1|specimen), family= Gamma(link=log),
                 data= data_crack_growth )

## GLMM with dispersion model##
res_glmm2 <- hglm2(y ~ crack0 + (1|specimen), family= Gamma(link=log),
                   data= data_crack_growth, disp= ~cycle )
```
library(dhglm)

## HGLM I ##

model_mu <- DHGLMMODELING(Model=\"mean\", Link=\"log\",
LinPred = y ~ crack0 + (1|specimen),
RandDist = \"inverse-gamma\")

model_phi <- DHGLMMODELING(Model=\"dispersion\")

res_hglm1 <- dhglmfit(RespDist=\"gamma\", DataMain=data_crack_growth,
MeanModel=model_mu, DispersionModel=model_phi)

## HGLM II ##

model_mu <- DHGLMMODELING(Model=\"mean\", Link=\"log\",
LinPred = y ~ crack0 + (1|specimen),
RandDist = \"inverse-gamma\")

model_phi <- DHGLMMODELING(Model = \"dispersion\", Link = \"log\",
LinPred = phi ~ cycle)

res_hglm2 <- dhglmfit(RespDist = \"gamma\", DataMain = data_crack_growth,
MeanModel = model_mu, DispersionModel = model_phi)
library(dhglm)

## HGLM I ##
model_mu <- DHGLMMODELING(Model="mean", Link="log",
    LinPred = y ~ crack0 + (1|specimen),
    RandDist = "inverse-gamma")
model_phi <- DHGLMMODELING(Model="dispersion")
res_hglm1 <- dhglmfit(RespDist="gamma", DataMain=data_crack_growth,
    MeanModel=model_mu, DispersionModel=model_phi)

## HGLM II ##
model_mu <- DHGLMMODELING(Model="mean", Link="log",
    LinPred = y ~ crack0 + (1|specimen),
    RandDist="inverse-gamma")
model_phi <- DHGLMMODELING(Model = "dispersion", Link = "log",
    LinPred = phi ~ cycle)
res_hglm2 <- dhglmfit(RespDist = "gamma", DataMain = data_crack_growth,
    MeanModel = model_mu, DispersionModelModel = model_phi)
Crack growth - Using the **dhglm** package

```r
## DHGLM I ##
model_mu <- DHGLMMODELING(Model="mean", Link="log",  
    LinPred = y ~ crack0 + (1|specimen), RandDist="inverse-gamma")

model_phi <- DHGLMMODELING(Model="dispersion", Link="log",  
    LinPred = phi ~ cycle + (1|specimen), RandDist="gaussian")

res_dhglm1 <- dhglmfit(RespDist = "gamma", DataMain = data_crack_growth,  
    MeanModel = model_mu, DispersionModel = model_phi)
```
Crack growth data - results

HGLM I

HGLM II

DHGLM I

Lee, Rönnegård & Noh

HGLM book

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Crack growth data - estimates

Estimates from the model(mu)

\[ y \sim \text{crack0} + (1 \mid \text{specimen}) \] [1] "log"

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>-5.654</td>
<td>0.07604</td>
<td>-74.36</td>
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<tr>
<td>crack0</td>
<td>2.364</td>
<td>0.05442</td>
<td>43.44</td>
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</table>

Estimates for logarithm of lambda=var(u_mu)

<table>
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<th>Estimate</th>
<th>Std. Error</th>
<th>t-value</th>
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</thead>
<tbody>
<tr>
<td>specimen</td>
<td>-3.385</td>
<td>0.3272</td>
<td>-10.34</td>
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</table>

Estimates from the model(phi)

\[ \phi \sim \text{cycle} + (1 \mid \text{specimen}) \] [1] "log"

<table>
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<th>Std. Error</th>
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<tr>
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<td>cycle</td>
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<td>2.2400</td>
<td>-4.566</td>
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</table>

Estimates for logarithm of var(u_phi)

<table>
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<th></th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>specimen</td>
<td>-1.099</td>
<td>0.2982</td>
<td>-3.684</td>
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</table>
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The hierarchical likelihood for observations $y$, fixed parameters $\theta$ and random effects $v$ is defined as

$$H(\theta, v; y) = f_\theta(y|v)f_\theta(v)$$

Inference of fixed effects: $\int H(\theta, v; y)dv$
Inference of random effects: $H(\theta, v; y)$
Inference of dispersion parameters: $f_\theta(y|\hat{\beta})$, i.e. REML
Here an example is presented to illustrate the fundamental idea of likelihood inference and how it may differ from Bayesian inference.

You walk down the street and come across a street magician. He has a small bag with a number of dice. There are two types of dice in the bag; white ones and blue ones. The white are numbered 1 to 6, while the blue have three sides with 1’s and three sides with 2’s. The magician draws a die at random from the bag without showing it to you and rolls the die. He claims that the number that turns up is a 2.

Which type of die would you guess he has rolled, a white or a blue one?
Instead of drawing a die, the street magician generated a random variable \( v \) from \( N(0,1) \). Suppose that he observed \( v = v_0 \). Then without telling the value of realized value \( v_0 \), he generates random variable \( Y \) from \( N(v_0,1) \). Then, he informs us the value \( y_0 \). Can we make an inference about \( v_0 \) given the observed data \( y_0 \)?
Development of books

1989
Generalized Linear Models
P. McCullagh and J.A. Nelder
CHAPMAN & HALL/CRC

2001
IN ALL LIKELIHOOD
Yudi Pawitan
Oxford

2006
Monographs on Statistics and Applied Probability 106
Generalized Linear Models with Random Effects
Unified Analysis via H-likelihood
Youngjo Lee, John A. Nelder, Yudi Pawitan
CHAPMAN & HALL/CRC

2017
DATA ANALYSIS USING HIERARCHICAL GENERALIZED LINEAR MODELS WITH R
Youngjo Lee, Lars Rönnegård, Meegaskol Noh
CRC Press

Lee, Rönnegård & Noh
HGLM book
Thank you!

All R code included in the book available at:

www.larsronnegard.se
Thank you!

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www.larsronnegard.se

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